Distributed Information-Lossless Space-Time Codes for Amplify-and-Forward TH-UWB Systems

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***Abstract*— In this paper, we extend the non-orthogonal amplify-and-forward (NAF) cooperative scheme [1] to the context of impulse radio ultra-wideband (UWB) systems. In particular, we consider the problem of distributed Space-Time (ST) coding with 2-dimensional Pulse Position Modulations (PPM) and Joint Pulse Position and Amplitude Modulations (PPAM) and we propose the first known family of full-rate codes that are information-lossless with these constellations. Being totally-real, these codes are adapted to the carrier-less nature of the UWB transmissions and they outperform all previously known totally-real constructions with any number of relays. With binary PPM, they satisfy all the construction constraints of the optimal complex-valued codes proposed in [2] as well as the additional constraint of being real-valued.**

***Index Terms*— UWB, Space-Time, AF, PAM, PPM.**

I. Introduction

RECENTLY Time-Hopping (TH) UWB WPANs (IEEE 802.15.3) have drawn considerable attention for short range radio links. On the other hand, cooperation diversity techniques [1]–[3] can boost the performance of such systems on which stringent transmission levels were imposed. In this context, the AF techniques can be appealing for UWB because of their simplicity (compared to other cooperation techniques).

In particular, the non-orthogonal AF strategy is known to achieve high performance levels with any number of relays [1]. Explicit codes that are optimal for the NAF scheme were proposed in [2]. However, being complex-valued, these codes are not suitable for low-cost carrier-less UWB transmissions. The construction techniques of [2] were adapted in [3] for the construction of totally-real codes for UWB systems based on cyclic division algebras (CDA). However, CDA-based codes can not be information-lossless when the construction must satisfy the additional constraint of being real-valued [3].

In this paper, we take advantage from the particular structure of the 2-dimensional 2-PPM and 2-PPM-M'-PAM constellations to construct totally-real information-lossless distributed ST codes for TH-UWB systems. These full-rate codes can achieve a full diversity order with any number of relays. Note that a similar approach was adopted in [3], however, the constructed codes were specific to constellations having higher dimensions and are, consequently, not adapted to the most popular UWB systems that use two modulation positions.

Notations: I_n is the $n \times n$ identity matrix. $0_{m \times n}$ corresponds to the $m \times n$ matrix whose elements are equal to 0. $\text{vec}(X)$ stacks the columns of the matrix X vertically. $\otimes$ corresponds to the Kronecker product.

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II. System Model

Consider the NAF protocol proposed in [1] with K relays. As in [2], [3], we consider the construction of minimal-delay codes. In this case, the k-th relay cooperates with the source during the k-th cooperation period that extends over 4 symbol durations. During the first half of this period, 2 encoded symbols are transmitted by the source. During the second half of this cooperation period, the source and the k-th relay transmit simultaneously. The source transmits 2 new encoded symbols while the k-th relay transmits amplified versions of the symbols that it received during the first half of the k-th cooperation period.

For TH-UWB systems where the information is modulated over M positions and M' amplitude levels, the decision matrix at the destination, when the NAF protocol is applied, can be written as:

$$Y_{(2LM \times 2K)} = \mathcal{H}_{(2LM \times 2KM)} C_{(2KM \times 2K)} + N_{(2LM \times 2K)} \quad (1)$$

where the subscripts indicate the corresponding matrices' dimensions and L is the number of fingers of the Rake receivers used at the relays and the destination. C is the distributed ST codeword.

The $[(i-1)LM + (l-1)M + m, 2(k-1) + j]$-th entry of the decision matrix Y corresponds to the decision variable collected at the m-th modulation position of the l-th Rake finger during the symbol duration $[4(k-1) + 2(i-1) + j]$ for $i = 1, 2$, $j = 1, 2$, $m = 1, \dots, M$, $l = 1, \dots, L$ and $k = 1, \dots, K$. N is the noise matrix that has the same structure as Y. It has a double-sided spectral density of $N_0/2$.

The channel matrix $\mathcal{H}$ is written as: $\mathcal{H} = [\mathcal{H}_1 \ \cdots \ \mathcal{H}_K]$ where $\mathcal{H}_k$ is a $2LM \times 2M$ matrix given by (for $k = 1, \dots, K$):

$$\mathcal{H}_k = \begin{bmatrix} \sqrt{\beta_1} H_0 & 0_{LM \times M} \\ \sqrt{\beta_1 \beta_2 \rho_k \lambda_k} \Sigma_k G_k \Psi_k H_k^T H_k & \sqrt{\beta_2} \Sigma_k H_0 \end{bmatrix} \quad (2)$$

where β_i determines the transmission level during the i-th half of each cooperation period for $i = 1, 2$. Normalizing the transmitted energy is obtained by fixing $\beta_1 + 2\beta_2 = 2$. ρ_k is a path-loss term corresponding to the quality of the channel between the source and the k-th relay while λ_k corresponds to the channel between the k-th relay and the destination. H_k is a $LM \times M$ matrix that corresponds to the channel between the source and the k-th relay for $k = 1, \dots, K$. The $((l-1)M + m, m')$-th element of H_k corresponds to the impact of the signal transmitted during the m'-th position on the m-th correlator placed after the l-th Rake finger for $l = 1, \dots, L$ and $m, m' = 1, \dots, M$. H_0 is the channel matrix between the source and the destination while G_k is the channel matrix between the k-th relay and the destination.

In order to avoid excessive channel delay spreads caused by the UWB channels, maximum ratio combining is applied at the relays before amplification and retransmission (this corresponds to the multiplication by H_k^T in eq. (2)). Ψ_k and Σ_k are the amplification and noise-whitening matrices respectively. They are given by:

$$\Psi_k = \left[H_k^T H_k \left(\beta_1 \rho_k H_k^T H_k + (N_0/2) I_M\right)\right]^{-\frac{1}{2}} \quad (3)$$

$$\Sigma_k = \left[I_{LM} + \beta_2 \lambda_k G_k \Psi_k H_k^T H_k \Psi_k^T G_k^T\right]^{-\frac{1}{2}} \quad (4)$$

III. CODE CONSTRUCTION

Each element of the hybrid M-PPM-M'-PAM constellation can be represented by a M-dimensional vector that belongs to the set:

$$\mathcal{C} = \{(2m'-1-M')e_m \ ; \ m' = 1\dots M' \ ; \ m = 1 \dots M\} \quad (5)$$

where e_m is the m-th column of I_M.

In what follows, we consider 2-dimensional constellations ($M = 2$). The most popular modulation schemes for TH-UWB are binary PPM and bi-orthogonal PPM and they follow as special cases by setting $M' = 1$ and $M' = 2$ respectively.

From eq. (1), the distributed ST coding scheme is determined by the codeword C given by:

$$C = \text{diag}\left(C_1 \ \cdots \ C_K\right) \quad (6)$$

where the cooperation between the source and the k-th relay is described by the $2M \times 2$ matrix C_k. The (m,i)-th (resp. $(M+m,i)$-th) entry of C_k corresponds to the amplitude of the pulse (if any) transmitted by the source (resp. k-th relay) at the m-th position of the $[4(k-1)+2+i]$-th symbol duration for $m = 1,\dots,M = 2$ and $i = 1,2$. Note that the symbols transmitted by the relay during the above symbol durations correspond to amplified versions of the symbols transmitted by the source during the symbol durations $4(k-1)+1$ and $4(k-1)+2$. The codeword C has a block diagonal structure because, in the NAF protocol, the relays do not transmit simultaneously.

We propose the following structure for the constituent sub-matrices:

$$C_k = \sigma^{k-1}\left(C_0\right) \quad (7)$$

$$C_0 = \begin{bmatrix} l_1 & l_2 \\ \Omega\tau(l_2) & \tau(l_1) \end{bmatrix} = \begin{bmatrix} k_1 + \phi k_2 & k_3 + \phi k_4 \\ \Omega(k_3 + \phi_1 k_4) & k_1 + \phi_1 k_2 \end{bmatrix} \quad (8)$$

where Ω is the 2×2 matrix given by:

$$\Omega = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad (9)$$

Let $\mathbb{K} = \mathbb{Q}(\theta)$ be a K-dimensional real cyclic field extension of $\mathbb{Q}$ and denote its Galois group by $Gal(\mathbb{K}/\mathbb{Q}) = \langle\sigma\rangle$ (with $\sigma^K = 1$). Then, from eq. (7), the codeword C_k transmitted by the source and the k-th relay during the second half of the k-th cooperation period corresponds to the $(k-1)$-th conjugate of C_0. In eq. (8), $k_1,\dots,k_4$ are M-dimensional vectors given by ($M = 2$):

$$k_i = \sum_{j=0}^{K-1} a_{(i-1)K+j+1}\theta^j \in \mathbb{K}^M \quad ; \quad i = 1,\dots,4 \quad (10)$$

where $a_1,\dots,a_{4K}$ are 2-dimensional vectors that belong to the 2-PPM-M'-PAM constellation given in eq. (5).

From eq. (7) and eq. (8), the entries of the codewords belong to the field $\mathbb{L}$ that is a 2-dimensional extension of $\mathbb{K}$: $\mathbb{L} = \mathbb{K}(\phi)$ where $\phi = \frac{1+\sqrt{5}}{2}$ is the golden number and $\phi_1 = \tau(\phi) = \frac{1-\sqrt{5}}{2}$. The Galois group of $\mathbb{L}$ is $\langle\tau\rangle$ with $\tau^2 = 1$.

Since $l_i \in \mathbb{L}^M = \mathbb{Q}^M(\theta,\phi)$ for $i = 1,2$, then $2K$ information symbols can be included in each value of l_1 or l_2. In other words, each codeword contains $4K$ information symbols resulting in no data rate reductions with respect to non-cooperative systems since the NAF scheme extends over $4K$ symbol durations.

Proposition 1: For a cooperative system with K relays, combining equations (6)-(9) permits to achieve a spatial diversity order of $K+1$ with 2-PPM-M'-PAM constellations for all values of K and M'.

Proof: Designate by $\Delta C(X,Y)$ the difference between two codewords C and C' that are associated with the information symbols $a_1,\dots,a_{4K}$ and $a'_1,\dots,a'_{4K}$ respectively. This matrix can be calculated from:

$$\Delta C(X,Y) = \text{diag}\left[\Delta C_0(X,Y) \ \cdots \ \sigma^{k-1}(\Delta C_0(X,Y))\right] \quad (11)$$

where:

$$\Delta C_0(X,Y) = C_0 - C'_0 = \begin{bmatrix} X & Y \\ \Omega\tau(Y) & \tau(X) \end{bmatrix} \quad (12)$$

where X and Y correspond to the difference between two elements of $\mathbb{L}^M$. They belong to the set:

$$\mathcal{A} = \left\{\sum_{i=1}^{2K}(c_i - c'_i)t_i \ ; \ c_1, c'_1, \dots, c_{2K}, c'_{2K} \in \mathcal{C}\right\} \subset \mathbb{L}^M \quad (13)$$

where $\{t_i\}_{i=1}^{2K} = \{1,\theta,\dots,\theta^{K-1},\phi,\phi\theta,\dots,\phi\theta^{K-1}\}$ and it forms a basis over $\mathbb{Q}^{2K}$ by construction. Consequently, $X = Y = 0_{2\times1}$ if and only if $a_i = a'_i$ for $i = 1,\dots,4K$. Therefore, the proposed code is fully diverse if $\Delta C(X,Y)$ has a full rank for $(X,Y) \in \mathcal{A}^2 \backslash \{(0_{2\times1}, 0_{2\times1})\}$. Following from eq. (11), this can happen only when $\Delta C_0(X,Y)$ is rank deficient since $\text{rank}[\sigma^{k-1}(\Delta C_0(X,Y))] = \text{rank}[\Delta C_0(X,Y)]$.

From eq. (12), $\text{rank}[\Delta C_0(X,Y)] < 2$ implies that there exists a non-zero constant $l \in \mathbb{L}$ such that: $Y = lX$ and $\tau(X) = l\Omega\tau(Y)$. Solving these equations, we obtain that X and Y must verify the equation:

$$\Omega X = \frac{1}{\text{N}_{\mathbb{L}/\mathbb{K}}(l)} X \quad (14)$$

showing that X and Y are eigenvectors of Ω. $\text{N}_{\mathbb{L}/\mathbb{K}}(l) \triangleq l\tau(l) \in \mathbb{K}$ is the algebraic norm of l.

The eigenvalues of the matrix Ω are equal to $\pm\sqrt{-1}$. Therefore, being real-valued, the non-zero vectors X and Y can not verify eq. (14). This shows that non-zero vectors X and Y that result in a rank deficient matrix $\Delta C_0(X,Y)$ do not belong to $\mathcal{A}$ given in eq. (13). Since the proof is independent from M', we conclude that the code achieves full diversity for all values of M'.

Proposition 2: For binary PPM, combining equations (6)-(9) permits to achieve the same coding gain as the optimal complex-valued codes proposed in [2].

Proof: For binary PPM, $\mathcal{C} = \{[1 \quad 0]^T, [0 \quad 1]^T\}$ and the difference between two elements of $\mathcal{C}$ belongs to the set $\{c[1 \ -1]^T \ ; \ c = 0, \pm1\}$. Therefore, X and Y can be written as $X=[x \quad -x]^T$ and $Y=[y \quad -y]^T$ where $x,y\in\mathbb{L}$. In this case, eq. (12) takes the following form:

$$\Delta C_0(X,Y)=\begin{bmatrix} x & -x & -\tau(y) & -\tau(y)\\ y & -y & \tau(x) & -\tau(x)\end{bmatrix}^T \tag{15}$$

This results in the following relation:

$$\begin{aligned}\det\left((\Delta C_0)^T\Delta C_0\right) &= \sum_{i=1}^{4}\sum_{j=i+1}^{4}\left(\det\left(\left[(\Delta C_{0,i})^T \quad (\Delta C_{0,j})^T\right]^T\right)\right)^2\\ &\geq \sum_{i=1}^{2}\sum_{j=3}^{4}\left(\det\left(\left[(\Delta C_{0,i})^T \quad (\Delta C_{0,j})^T\right]^T\right)\right)^2\\ &= 4\left(\left(\mathrm{N}_{\mathbb{L}/\mathbb{K}}(x)\right)^2+\left(\mathrm{N}_{\mathbb{L}/\mathbb{K}}(y)\right)^2\right) \end{aligned}\tag{16}$$

where $\Delta C_{0,i}$ is the i-th row of ΔC_0. Following from eq. (11):

$$\det\left((\Delta C)^T\Delta C\right)\geq 4^K\left(\left(\mathrm{N}_{\mathbb{L}/\mathbb{Q}}(x)\right)^2+\left(\mathrm{N}_{\mathbb{L}/\mathbb{Q}}(y)\right)^2\right) \tag{17}$$

Therefore, the minimum non-zero value of eq. (17) is equal to 4^K since $\mathrm{N}_{\mathbb{L}/\mathbb{Q}}(x)\in\mathbb{Z}$ and $\mathrm{N}_{\mathbb{L}/\mathbb{Q}}(y)\in\mathbb{Z}$ given that x and y are constrained to belong to the ring of integers of $\mathbb{L}$.

Proposition 3: The proposed code is information-lossless.

Proof: According to the definition given in [4], the code is information-lossless if the transmitted encoded multi-dimensional constellation is a rotated version of the information constellation.

Designate by Φ the $4KM\times 4KM$ matrix that verifies the following relation ($M=2$):

$$\mathrm{vec}\left(C'\right)\triangleq\mathrm{vec}\left(\left[C_1^T \ \cdots \ C_K^T\right]^T\right)=\Phi[a_1^T \ \cdots \ a_{4K}^T]^T \tag{18}$$

In other words, C' is the vertical concatenation of the 2×2 matrices $C_1,\dots,C_K$ given in eq. (7). In this case, Φ determines the linear dependence between the encoded symbols and the information symbols $a_1,\dots,a_{4K}$.

From eq. (7) and eq. (8), it is straight-forward to verify that the matrix Φ is given by:

$$\Phi=\begin{bmatrix}\Phi_1^T & \Phi_2^T\end{bmatrix}^T \tag{19}$$

$$\Phi_i=\begin{bmatrix}\Phi_{i,0}^T & \cdots & \sigma^{K-1}(\Phi_{i,0}^T)\end{bmatrix}^T \quad ; \quad i=1,2 \tag{20}$$

$$\Phi_{1,0}=\begin{bmatrix}\mathcal{M} & \phi\mathcal{M} & 0_{M\times KM} & 0_{M\times KM}\\ 0_{M\times KM} & 0_{M\times KM} & \Omega\mathcal{M} & \phi_1\Omega\mathcal{M}\end{bmatrix} \tag{21}$$

$$\Phi_{2,0}=\begin{bmatrix}0_{M\times KM} & 0_{M\times KM} & \mathcal{M} & \phi\mathcal{M}\\ \mathcal{M} & \phi_1\mathcal{M} & 0_{M\times KM} & 0_{M\times KM}\end{bmatrix} \tag{22}$$

where $\mathcal{M}$ is the $M\times KM$ matrix given by:

$$\mathcal{M}\triangleq\mathcal{M}_0\otimes I_M=\begin{bmatrix}1 & \theta & \cdots & \theta^{K-1}\end{bmatrix}\otimes I_M$$

From [4], the code is information-lossless if Φ is unitary. Since Ω is unitary, then Φ can be made unitary when the two basis $\{\theta^i\}_{i=0}^{K-1}$ and $\{\phi^i\}_{i=0}^{1}$ are replaced by new totally-real orthonormal basis $\{\sigma^i(v)\}_{i=0}^{K-1}$ and $\{\tau^i(u)\}_{i=0}^{1}$. u is given by $u=(\frac{3-\phi}{5})^{1/2}$ while v depends on the number of relays. For example, with $K=2$ relays, v can be chosen as $v=\frac{\sqrt{3-\theta}}{2}$ where $\theta=1+\sqrt{2}$. More details on the construction of these basis can be found in [3].

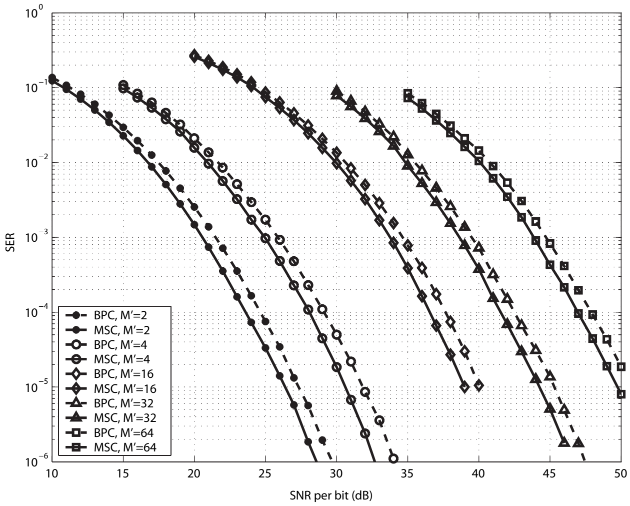

Fig. 1. The proposed modulation-specific code (MSC) vs. the best previously known totally-real code (BPC) [3] with one relay and 2-PPM-M'-PAM.

IV. Simulations and Results

Simulations are performed over the IEEE 802.15.3a channel model recommendation CM1 [5]. Fig. 1 compares the proposed modulation-specific code (MSC) with the best previously known totally-real code (BPC) [3] with one relay and a 5-finger Rake. We fix $\beta_1=\beta_2$ and $\rho_1=\lambda_1=1$ in eq. (2). While both families of codes achieve full rate and full diversity, BPC presents the advantage of having a non-vanishing coding gain while MSC is information-lossless. For the latter codes, there is no explicit expression of the coding gain given that the determinants of the codewords are not integers. We might imagine that BPC will outperform MSC for large values of M' since the coding gain of the former remains constant. However, the results in Fig. 1 show the superiority of the modulation-specific codes even at very high spectral efficiencies. Similar results are obtained for larger numbers of relays.

V. Conclusion

We investigated the problem of constructing distributed ST codes suitable for the NAF strategy with 2-PPM-M'-PAM constellations. We presented new totally-real constructions that are suitable for carrier-less cooperative UWB systems with any number of relays. These constructions solve the problem of the nonexistence of information-lossless and totally-real constructions. They outperform the best known totally-real distributed ST codes based on cyclic division algebras.

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